Accurate Solutions of Elliptical and Cylindrical Striplines and Microstrip Lines

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Abstract —**By the transformation methods, elliptical and cylindrical striplines and microstrip lines are theoretically analyzed. Rigorous derivations lead to the exact expressions of the characteristic impedance in closed-form. Elliptical microstrip lines are explored for the first time. For practical applications, elliptical and cylindrical striplines with finite thickness are also analyzed.**

I. Introduction

FOR APPLICATIONS such as the transition adapter, balun, slotted line, etc., a number of investigations on elliptical and cylindrical striplines and cylindrical microstirp lines have been reported. For cylindrical striplines and microstrip lines with the dual series, two methods, i.e., the least-square and the simple integration methods, are used to solve constants appearing in the series [1]. For elliptical and cylindrical striplines, by separating the variables, the solution of the two-dimensional Laplace's equation in orthogonal curvilinear elliptical coordinates is expressed in the form of a series [2]. Then the constants of solution are determined by the modified residue calculas technique (MRCT) developed by Mitta [3]. The variational expression of lines can be obtained by Green's function method [4]. The derivations in these analyses, however, are not rigorous, and the solutions are approximate because of the series used. At the same time, elliptical microstrip lines have not yet been explored. In this paper, four kinds of transmission lines, i.e., elliptical and circular cylindrical striplines and microstrip lines, are rigorously analyzed with the conformal mapping technique [5]. By the transformation methods, the elliptical or circular cylindrical striplines are transformed into asymmetric or symmetric planar striplines, and the elliptical or circular cylindrical microstrip lines are transformed into planar microstrip lines, which are readily analyzed. Based on these rigorous derivations, exact expressions of the characteristic impedance for these transmission lines are presented. For practical applications, elliptical and cylindrical striplines with finite thicknesses greater than zero are also analyzed or the first time.

II. Elliptical and Cylindrical Striplines

For a cross section of an elliptical stripline shown in Fig. 1, there are three confocal ellipses, S_1, S_2, and S_3. Among them, two elliptical cylinders, S_1 and S_3, are grounded.

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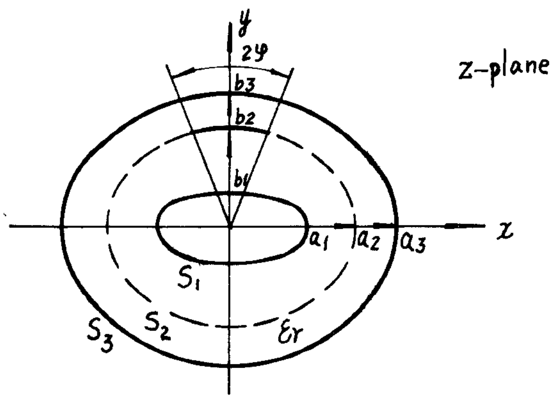

Their semi-major axes and semi-minor axes are a_1, a_2, a_3, b_1, b_2, and b_3, respectively. An angle subtended by the arc-strip at center is 2φ. The dimensional relationship between the three ellipses is

$$c=\sqrt{a_1^2-b_1^2}=\sqrt{a_2^2-b_2^2}=\sqrt{a_3^2-b_3^2}. \tag{1}$$

We shall assume that only TEM modes exist. In this case, consider the transformation function

$$\zeta=\left(z\pm\sqrt{z^2-c^2}\right)/c \tag{2}$$

where

$$\zeta=\xi+j\eta \quad z=x+jy.$$

Using this transformation function, to can be shown that ellipses S_1, S_2, and S_3 in the z plane are transformed into circles S_1', S_2', and S_3' in the ζ plane, respectively, as shown in Fig. 2. Their radii are given by

$$r_1=\sqrt{(a_1+b_1)/(a_1-b_1)}$$
$$r_2=\sqrt{(a_2+b_2)/(a_2-b_2)}$$
$$r_3=\sqrt{(a_3+b_3)/(a_3-b_3)}.$$

In this way, the elliptical stripline has been transformed into an cylindrical stripline with the arc-strip length

$$d=2\varphi\cdot r_2. \tag{3}$$

With the transformation function

$$w=j\ln\zeta+\frac{\pi}{2} \tag{4}$$

$$w=u+jv \quad \zeta=\xi+j\eta \tag{5}$$

this cylindrical stripline is transformed into an asymmetric

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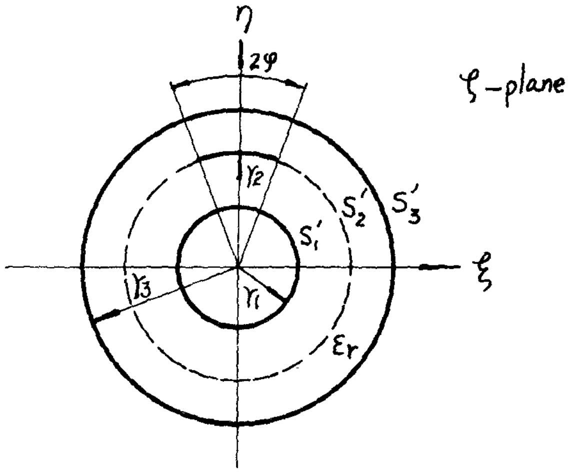

Fig. 2. Cross section of cylindrical stripline.

planar stripline, as shown in Fig. 3. The relationships between the two striplines are given by

$$v_1 = \ln r_1$$
$$v_2 = \ln r_2$$
$$v_3 = \ln r_3. \tag{6}$$

Assuming that the asymmetric planar stripline has negligible thickness, the width of the center conductor is

$$W = 2\varphi. \tag{7a}$$

The distance between the two ground plates is

$$H = \ln(r_3/r_1) \tag{7b}$$

and the distance from the center conductor to the lower ground plate is

$$h = \ln(r_2/r_1). \tag{7c}$$

So far, the elliptical and cylindrical striplines have been transformed into the asymmetric planar stripline, which has been analyzed by conformal mapping in the literature [6]. Its exact expression of the characteristic impedance is given by

$$Z_0 = \frac{29.976\pi}{\sqrt{\epsilon_r}} \frac{K'(k)}{K(k)} \tag{8}$$

where $K(k)$ is the complete elliptical integral of the first kind, and k and k' are the moduli. The moduli are given by [7]

$$k = \left(\frac{e^{\pi K(k)/K'(k)} - 2}{e^{\pi K(k)/K'(k)} + 2}\right)^2, \qquad 1 \leqslant \frac{K(k)}{K'(k)} \leqslant \infty \tag{9a}$$

$$k' = \left(\frac{e^{\pi K'(k)/K(k)} - 2}{e^{\pi K'(k)/K(k)} + 2}\right)^2, \qquad 0 \leqslant \frac{K(k)}{K'(k)} \leqslant 1 \tag{9b}$$

$$k' = (1-k^2)^{1/2}, \qquad 0 \leqslant k \leqslant 1.$$

The parameters of the elliptical stripline can be determined

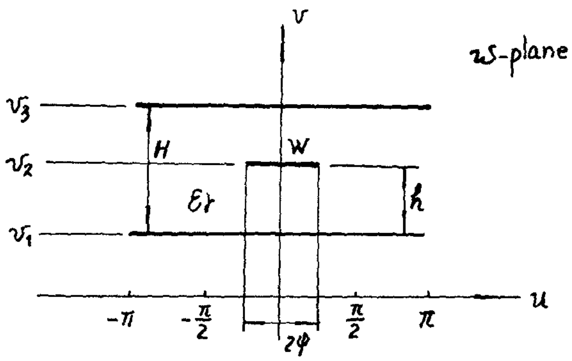

Fig. 3. Cross section of asymmetric planar stripline.

by the following set of equations:

$$A = \frac{K(k)}{\Pi(\lambda, K(k), k)} \tag{10a}$$

$$B = \frac{1}{4\pi} \ln\left[\frac{(a_3+b_3)(a_1-b_1)}{(a_3-b_3)(a_1+b_1)}\right] \cdot \ln\left[\frac{k-1}{2\lambda - k - 1 + 2\sqrt{(1-\lambda)(k-\lambda)}}\right] + j\frac{1}{2} \ln\left[\frac{(a_2+b_2)(a_1-b_1)}{(a_2-b_2)(a_1+b_1)}\right] \tag{10b}$$

$$C = -\frac{1}{2\pi} \frac{\sqrt{(1-\lambda)(k-\lambda)}}{A\sqrt{\lambda}} \ln\left[\frac{(a_3+b_3)(a_1-b_1)}{(a_3-b_3)(a_1+b_1)}\right] \tag{10c}$$

$$\frac{F(\sin^{-1}\alpha, k)}{K(k)} = 1 - 2\left\{\ln\left[\frac{(a_2+b_2)(a_1-b_1)}{(a_2-b_2)(a_1+b_1)}\right] \Big/ \ln\left[\frac{(a_3+b_3)(a_1+b_1)}{(a_3-b_3)(a_1+b_1)}\right]\right\} \tag{10d}$$

$$2\varphi = \left\{\frac{\sqrt{(1-\lambda)(k-\lambda)}}{\pi A\sqrt{\lambda}} \cdot \left[F(\sin^{-1}\beta, k) - A\Pi(\lambda, \sin^{-1}\beta, k)\right] - \frac{1}{2\pi} \ln\left[\frac{\sqrt{(1-\lambda)(1-k\beta^2)} - \sqrt{(k-\lambda)(1-\beta^2)}}{\sqrt{(1-\lambda)(1-k\beta^2)} + \sqrt{(k-\lambda)(1-\beta^2)}}\right]\right\} \cdot \ln\left[\frac{(a_3+b_3)(a_1-b_1)}{(a_3-b_3)(a_1+b_1)}\right] \tag{10e}$$

$$\alpha = \sqrt{\lambda/k} \quad \beta = (1-A)/\sqrt{\lambda}. \tag{10f}$$

Here, $F(\sin^{-1}\alpha, k)$ and $F(\sin^{-1}\beta, k)$ are the incomplete elliptical integrals of the fist kind, $\Pi(\lambda, \sin^{-1}\beta, k)$ is the incomplete elliptical integral of the third kind, and $\Pi(\lambda, K(k), k)$ is the complete elliptical integral of the third kind. The relative error of calculation with (9) is less than $4 \cdot 10^{-12}$. Similarly, the characteristic impedance of the cylindrical stripline can be obtained

$$Z_0=\frac{29.976\pi}{\sqrt{\epsilon_r}}\frac{K'(k)}{K(k)} \tag{11}$$

$$A=\frac{K(k)}{\Pi(\lambda, K(k), k)} \tag{12a}$$

$$B=\frac{1}{2\pi}\ln\left(\frac{r_3}{r_1}\right)\cdot\ln\left[\frac{k-1}{2\lambda-k-1+2\sqrt{(1-\lambda)(k-\lambda)}}\right]+j\ln\left(\frac{r_2}{r_1}\right) \tag{12b}$$

$$C=-\frac{1}{\pi}\frac{\sqrt{(1-\lambda)(k+\lambda)}}{A\sqrt{\lambda}}\ln\left(\frac{r_3}{r_1}\right) \tag{12c}$$

$$\frac{F(\sin^{-1}\alpha, k)}{K(k)}=1-2\left[\ln\left(\frac{r_2}{r_1}\right)\Big/\ln\left(\frac{r_3}{r_1}\right)\right] \tag{12d}$$

$$\varphi=\left\{\frac{\sqrt{(1-\lambda)(k-\lambda)}}{\pi A\sqrt{\lambda}}\cdot\left[F(\sin^{-1}\beta, k)-A\Pi(\lambda, \sin^{-1}\beta, k)\right]-\frac{1}{2\pi}\ln\left[\frac{\sqrt{(1-\lambda)(1-k\beta^2)}-\sqrt{(k-\lambda)(1-\beta^2)}}{\sqrt{(1-\lambda)(1-k\beta^2)}+\sqrt{(k-\lambda)(1-\beta^2)}}\right]\right\}\cdot\ln\left(\frac{r_3}{r_1}\right). \tag{12e}$$

For a given Z_0, substituting (8) or (11) into (9), the modulus k can be calculated with (9). λ can be obtained from (10f) and (10d) or (12d). The constants A, B, and C are determined by (10a), (10b), (10c), or (12a), (12b), and (12c), respectively. Finally, the angle subtended by arc-strip 2φ is calculated from (10e) or (12e); then the width of the center conductor is readily obtained.

Through a similar rigorous derivation, it can be shown that when the elliptical stripline satisfies

$$\frac{a_2+b_2}{a_2-b_2}=\sqrt{\frac{(a_3+b_3)(a_1+b_1)}{(a_3-b_3)(a_1-b_1)}} \tag{13}$$

it is transformed into a symmetric planar stripline, for which the relatively exact expression of the characteristic impedance has been given by Cohn [8]. In this case, the relatively simple and exact expression of the characteristic impedance is

$$Z_0=\frac{29.976\pi}{\sqrt{\epsilon_r}}\frac{K(k)}{K'(k)} \tag{14}$$

$$k=\operatorname{sech}\left\{2\varphi\pi/\ln\left[\frac{(a_3+b_3)(a_1-b_1)}{(a_3-b_3)(a_1+b_1)}\right]\right\}. \tag{15}$$

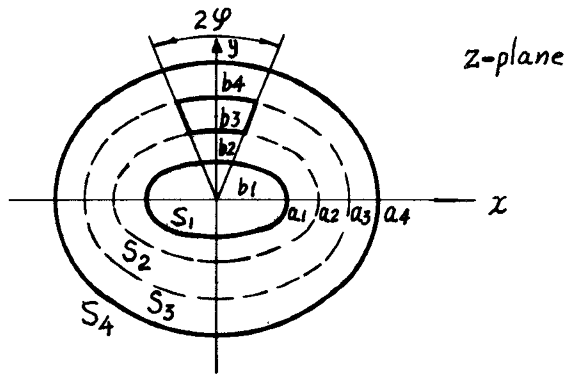

Fig. 4. Cross section of elliptical stripline with finite thickness.

In the same way, under the condition

$$r_2=\sqrt{r_1\cdot r_3} \tag{16}$$

the cylindrical stripline is transformed into a symmetric planar stripline, and its relatively simple and exact expression of the characteristic impedance is

$$Z_0=\frac{29.976\pi}{\sqrt{\epsilon_r}}\frac{K(k)}{K'(k)} \tag{17}$$

$$k=\operatorname{sech}\left[\varphi\pi/\ln\left(\frac{r_3}{r_1}\right)\right]. \tag{18}$$

In order to simplify the calculations of (14) and (17), we can use the following [7]:

$$\frac{K(k)}{K'(k)}=\frac{1}{2\pi}\ln\left[2\frac{(1+k)^{1/2}+(4k)^{1/4}}{(1+k)^{1/2}-(4k)^{1/4}}\right],\quad \frac{1}{\sqrt{2}}\leqslant k\leqslant 1 \tag{19a}$$

$$\frac{K(k)}{K'(k)}=2\pi\Big/\ln\left[2\frac{(1+k')^{1/2}+(4k')^{1/4}}{(1+k')^{1/2}-(4k')^{1/4}}\right],\quad 0\leqslant k\leqslant\frac{1}{\sqrt{2}} \tag{19b}$$

$$k'=(1-k^2)^{1/2}. \tag{19c}$$

From (13)–(18), it can be shown that when (13) and (16) are met, calculations of (14) and (17) for the characteristic impedance are not only simple but also relatively accurate, which is valuable for practical applications.

III. Elliptical and Cylindrical Striplines with Finite Thickness

The analyses mentioned above neglect the effect of strip thickness. Elliptical and cylindrical striplines with finite thickness greater than zero can be analyzed as follows.

A. *Elliptical Striplines with Finite Thickness*

In this case, the center conductor of elliptical stripline consists of two lengths of arc-strip, subtending an angle 2φ of ellipses S_2 and S_3, as shown in Fig. 4. Four ellipses, S_1,

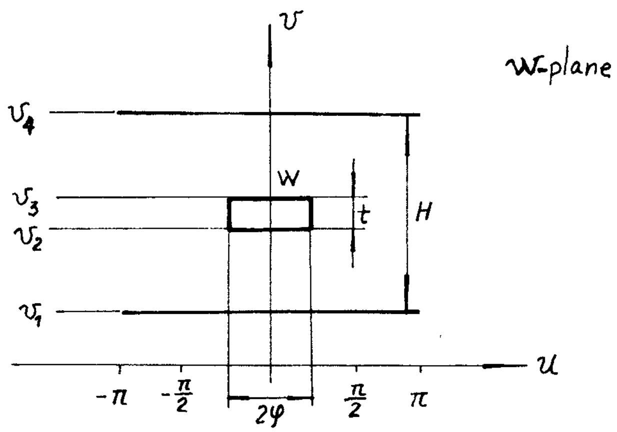

Fig. 5. Cross section of symmetric planar stripline with finite thickness.

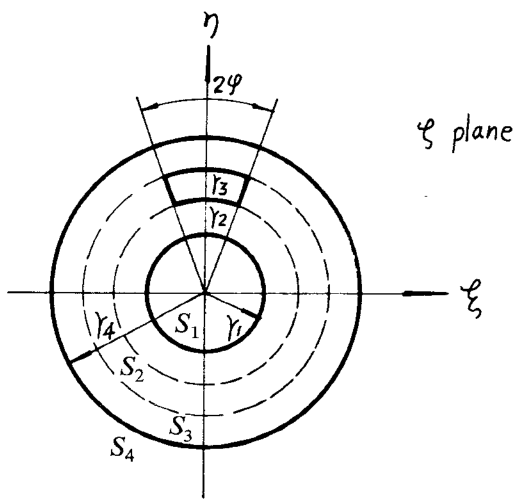

Fig. 6. Cross section of cylindrical stripline with finite thickness.

S_2, S_3, and S_4, are confocal, and elliptical cylinders S_1 and S_4 are grounded. The thickness of the center conductor is $(b_3 - b_2)$.

With the transformation function

$$W = j\operatorname{arc\,ch}(z/c) + \frac{\pi}{2}$$

$$W = u + jv \quad z = x + jy \tag{20}$$

the elliptical stripline with finite thickness is transformed into an asymmetric planar stripline with finite thickness. It can be shown that the relationships between the two strip-lines are

$$v_1 = \frac{1}{2}\ln\left[(a_1+b_1)/(a_1-b_1)\right]$$

$$v_2 = \frac{1}{2}\ln\left[(a_2+b_2)/(a_2-b_2)\right]$$

$$v_3 = \frac{1}{2}\ln\left[(a_3+b_3)/(a_3-b_3)\right]$$

$$v_4 = \frac{1}{2}\ln\left[(a_4+b_4)/(a_4-b_4)\right].$$

When this elliptical stripline satisfies

$$\frac{(a_4+b_4)(a_3-b_3)}{(a_4-b_4)(a_3+b_3)} = \frac{(a_2+b_2)(a_1-b_1)}{(a_2-b_2)(a_1+b_1)} \tag{21}$$

the corresponding asymmetric planar stripline with finite thickness becomes a symmetric planar stripline, as shown in Fig. 5. The thickness and width of its center conductor are given by

$$t = \frac{1}{2}\ln\frac{(a_3+b_3)(a_2-b_2)}{(a_3-b_3)(a_2-b_2)} \tag{22}$$

$$W = 2\varphi. \tag{23}$$

The distance between the two plates is given by

$$H = \frac{1}{2}\ln\frac{(a_4+b_4)(a_1-b_1)}{(a_4-b_4)(a_1+b_1)}. \tag{24}$$

For this situation, the exact expressions of the characteristic impedance in the form of an implicit function have been reported by Waldron [9]. So

$$Z_0 = \frac{29.976\pi\, K'(1/m)}{\sqrt{\epsilon_r}\, K(1/m)} \tag{25}$$

$$\ln\left[\frac{(a_3+b_3)(a_2-b_2)}{(a_3-b_3)(a_2+b_2)}\right] \Big/ \ln\left[\frac{(a_4+b_4)(a_1-b_1)}{(a_4-b_4)(a_1+b_1)}\right] = \frac{-\left[K(n') - R\Pi(R', n')\right]}{R\left[\Pi(R', n) + \Pi(1-m^2, n') - K(n')\right]} \tag{26a}$$

$$4\varphi \Big/ \ln\left[\frac{(a_4+b_4)(a_1-b_1)}{(a_4-b_4)(a_1+b_1)}\right] = \frac{K(n) - (1-n^2/m^2)\Pi(n^2/m^2, n)}{R\left[\Pi(R', n) + \Pi(1-m^2, n') - K(n')\right]} \tag{26b}$$

$$R = \frac{m^2-n^2}{m^2-1} \quad R' = \frac{1-n^2}{1-m^2} \quad n' = \sqrt{1-n^2} \tag{26c}$$

where K and Π are the complete elliptical integrals of the first and third kinds, respectively.

Equation (25) and (26) are exact but difficult to calculate. In practice, the approximate formulas with a relative error of one percent are used [10]

$$Z_0 \doteqdot \frac{94.172}{\sqrt{\epsilon_r}\, x(W/H) + (1/\pi)P(x)} \tag{27}$$

$$P(x) = \ln\frac{(x+1)^{x+1}}{(x-1)^{x-1}} \quad x = \frac{1}{1-t/H} \tag{28a}$$

$$W/H \geqslant 0.35(1-t/H) \quad t/H \leqslant 0.25. \tag{28b}$$

B. Cylindrical Stripline with Finite Thickness

In this case, the center conductor of the cylindrical stripline consists of the two lengths of arc-strip, subtending an angle 2φ of cylinders S_2 and S_3, as shown in Fig. 6. The thickness of its center conductor is $(r_3 - r_2)$.

With the transformation function (eq. (4)), this cylindrical stripline is transformed into an asymmetric planar

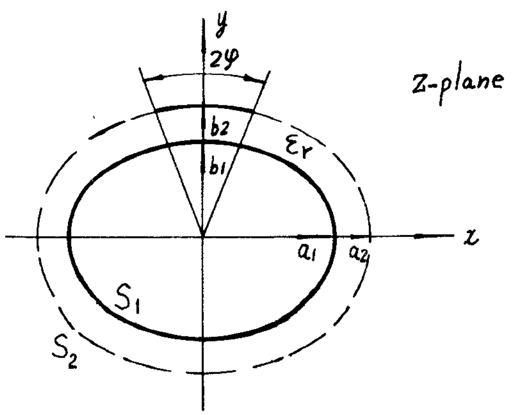

Fig. 7. Cross section of elliptical microstrip line.

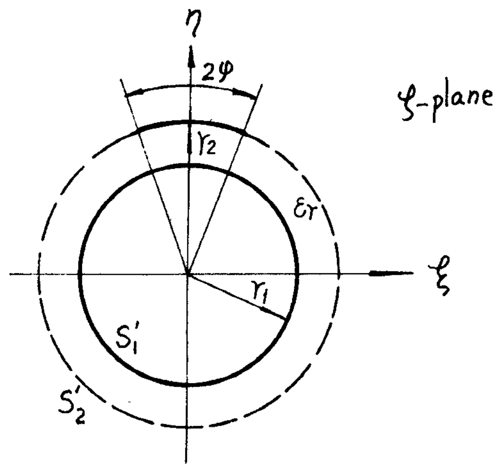

Fig. 8. Cross section of cylindrical microstrip line.

stripline with finite thickness. When this cylindrical stripline satisfies

$$r_4 \cdot r_1 = r_2 \cdot r_3 \tag{29}$$

the asymmetric planar stripline becomes a symmetric planar stripline with finite thickness, which is the same as shown in Fig. 5. The thickness and width of its center conductor are

$$t = \ln(r_3/r_2) \tag{30}$$

$$W = 2\varphi. \tag{31}$$

The distance between the two plates is

$$H = \ln(r_4/r_1). \tag{32}$$

Similarly, the exact expressions of the characteristic impedance for cylindrical striplines are given by

$$Z_0 = \frac{29.976\pi K'(1/m)}{\sqrt{\epsilon_r}\, K(1/m)} \tag{33}$$

$$\frac{\ln(r_3/r_2)}{\ln(r_4/r_1)} = \frac{-[K(n') - R\Pi(R', n)]}{R[\Pi(R', n) + \Pi(1-m^2, n') - K(n')]} \tag{34a}$$

$$\frac{2\varphi}{\ln(r_4/r_1)} = \frac{K(n) - (1 - n^2/m^2)\Pi(n^2/m^2, n)}{R[\Pi(R', n) + \Pi(1-m^2, n') - K(n')]}. \tag{34b}$$

In order to simplify the calculations, (30)–(32) can be substituted into (27) and (28). Then the obtained results have the relative error of order of one percent.

IV. Elliptical and Cylindrical Microstrip Lines

A cross section of an elliptical microstrip line is shown in Fig. 7. The angle subtended by its arc-strip at the center is 2φ, and the elliptical cylinder S_1 is grounded. ϵ_r is the relative dielectric constant of the substrate.

For this transmission line, the same method as mentioned above can be used. Assuming that only TEM modes exist, with the transformation function (eq. (2)), ellipses S_1 and S_2 in the z plane are transformed into circles S_1' and S_2' in the ζ plane, as shown in Fig. 8. The radii of circles are

$$r_1 = \sqrt{(a_1 + b_1)/(a_1 - b_1)} \tag{35a}$$

and

$$r_2 = \sqrt{(a_2 + b_2)/(a_2 - b_2)}. \tag{35b}$$

So the elliptical microstrip line has been transformed into an cylindrical microstrip line with the width of its center conductor

$$d = 2\varphi r_2. \tag{36}$$

With the transformation function (eq. (4)), the cylindrical microstrip line is transformed into a planar microstrip line as shown in Fig. 9. The relationships between the two transmission lines are as follows:

$$v_1 = \ln\, r_1 \tag{37a}$$

$$v_2 = \ln\, r_2. \tag{37b}$$

For the planar microstrip line, the width of its center conductor is

$$W = 2\varphi \tag{38}$$

and the distance between its center conductor and ground plate, i.e., the thickness of its substrate, is given by

$$h = \ln(r_2/r_1). \tag{39}$$

With the method proposed by Wheeler [11], its relatively exact expressions of characteristic impedance can be obtained as follows.

1) For Narrow Strips:

$$\left\{4\varphi \Big/ \ln\left[\frac{(a_2+b_2)(a_1-b_1)}{(a_2-b_2)(a_1+b_1)}\right]\right\} < 1$$

$$Z_0 = \frac{376.687}{\pi\sqrt{2(\epsilon_r+1)}}\left\{\ln\left(\frac{2}{\varphi}\, ln\left[\frac{(a_2+b_2)(a_1-b_1)}{(a_2-b_2)(a_1+b_1)}\right]\right) + \frac{1}{32}\left(4\varphi \Big/ \ln\left[\frac{(a_2+b_2)(a_1-b_1)}{(a_2-b_2)(a_1+b_1)}\right]\right)^2 - \frac{1}{2}\left(\frac{\epsilon_r-1}{\epsilon_r+1}\right)\left(\ln\frac{\pi}{2} + \frac{1}{\epsilon_r}\ln\frac{4}{\pi}\right)\right\}. \tag{40}$$

2) For Wide Strips:

$$\left\{4\varphi \Big/ \ln\left[\frac{(a_2+b_2)(a_1-b_1)}{(a_2-b_2)(a_1+b_1)}\right]\right\} > 1$$

$$Z_0 = \frac{376.687}{2\sqrt{\epsilon_r}}\left\{2\varphi \Big/ \ln\left[\frac{(a_2+b_2)(a_1-b_1)}{(a_2-b_2)(a_1+b_1)}\right] + 0.441 + 0.082\left(\frac{\epsilon_r-1}{\epsilon_r^2}\right) + \left(\frac{\epsilon_r+1}{2\pi\epsilon_r}\right)\left[1.451 + \ln\left(2\varphi \Big/ \ln\left[\frac{(a_2+b_2)(a_1-b_1)}{(a_2-b_2)(a_1+b_1)}\right] + 0.94\right)\right]\right\}^{-1} \quad (41)$$

and the effective dielectrical constant of its substrate

$$\epsilon_e = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2} \cdot \left\{1 + \frac{5}{2\varphi}\ln\left[\frac{(a_2+b_2)(a_1-b_1)}{(a_2-b_2)(a_1+b_1)}\right]\right\}^{-1/2}. \quad (42)$$

In the same way, the characteristic impedance of cylindrical microstrip lines can be calculated as follows.

1) For Narrow Strips:

$$[2\varphi/\ln(r_2/r_1)] < 1$$

$$Z_0 = \frac{376.687}{\pi\sqrt{2(\epsilon_r+1)}}\left\{\ln\left[\frac{4}{\varphi}\ln\left(\frac{r_2}{r_1}\right)\right] + \frac{1}{32}\left[2\varphi \Big/ \ln\left(\frac{r_2}{r_1}\right)\right]^2 - \frac{1}{2}\left(\frac{\epsilon_r-1}{\epsilon_r+1}\right) \cdot \left(\ln\frac{\pi}{2} + \frac{1}{\epsilon_r}\ln\frac{4}{\pi}\right)\right\}. \quad (43)$$

2) For Wide Strips:

$$[2\varphi/\ln(r_2/r_1)] > 1$$

$$Z_0 = \frac{376.687}{2\sqrt{\epsilon_r}}\left\{\varphi \Big/ \ln\left(\frac{r_2}{r_1}\right) + 0.441 + 0.082\left(\frac{\epsilon_r-1}{\epsilon_r^2}\right) + \left(\frac{\epsilon_r+1}{2\pi\epsilon_r}\right) \cdot \left(1.451 + \ln\left[\varphi \Big/ \ln\left(\frac{r_2}{r_1}\right) + 0.94\right]\right)\right\}^{-1} \quad (44)$$

and the effective dielectric constant of its substrate

$$\epsilon_e = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\left[1 + \frac{5}{\varphi}\ln\left(\frac{r_2}{r_1}\right)\right]^{-1/2}. \quad (45)$$

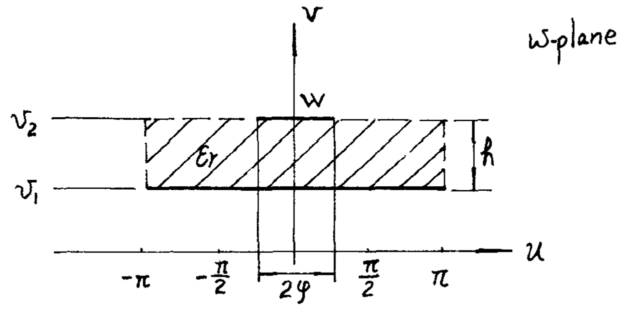

Fig. 9. Cross section of planar microstrip line.

The relative error of the calculations with (42) and (45) is less than one percent.

V. Conclusions

In the preceding sections, elliptical and cylindrical striplines are rigorously transformed into asymmetric planar striplines and the analysis leads to exact expressions for the characteristic impedance (eqs. (8) and (11)). For the particular cases, the derived expressions (eqs. (14), (15), (17), and (18)) are both accurate and simple for calculating the characteristic impedance, which will be, therefore, of practical interest. In the case of strip thickness greater than zero, the corresponding expressions are presented. The derived formulas of the characteristic impedance for elliptical and cylindrical microstrip lines are not only simple to calculate, but also relatively accurate.

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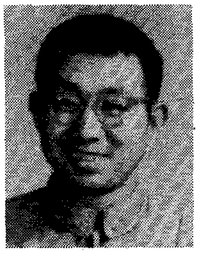

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